$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$$

The matrix is triangular (in fact diagonal in this case) so the eigenvalues are given by the leading diagonal line.

If you are unsure about this then just use the regular eigenvalue equation to solve for λ.

$\lambda = 1$:

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The system of equations this represents is:

$$0x_1 + x_2 = 0$$
$$0x_1 + 0x_2 = 0$$

The first equation means that x_2 must be 0.

Both equations allow x_1 to be anything since it is multiplied by 0.

The solution would then be,

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

We just choose any easy value of x_1 to get one eigenvector. So letting $x_1 = 1$ an eigenvector is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.