

$$A = \begin{pmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{pmatrix}$$

$$|A - \lambda x| = \begin{vmatrix} 3 - \lambda & 2 & -3 \\ -3 & -4 - \lambda & 9 \\ -1 & -2 & 5 - \lambda \end{vmatrix} = -x^3 + 4x^2 - 4x = -x(x - 2)^2$$

$$\lambda = 0, 2, 2$$

$\lambda = 0$:

$$\begin{pmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow x = x_3 \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$$

$\lambda = 2$

$$\begin{pmatrix} 1 & 2 & -3 \\ -3 & -6 & 9 \\ -1 & -2 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow x = x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} -1 & 3 & -2 \\ 3 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$P^{-1} = \begin{pmatrix} 1/2 & 1 & -3/2 \\ -1/2 & -1 & 5/2 \\ -3/2 & -2 & 9/2 \end{pmatrix}$$

$$B = P^{-1}AP = \begin{pmatrix} 1/2 & 1 & -3/2 \\ -1/2 & -1 & 5/2 \\ -3/2 & -2 & 9/2 \end{pmatrix} \begin{pmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{pmatrix} \begin{pmatrix} -1 & -2 & 3 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & 1 & -3/2 \\ -1/2 & -1 & 5/2 \\ -3/2 & -2 & 9/2 \end{pmatrix} \begin{pmatrix} 0 & 6 & -4 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$