$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

This matrix represents the homogeneous matrix equation,

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Writing it out gives,

$$\begin{aligned} 0x_1 + x_2 + x_3 &= 0 \\ 0x_1 + 0x_2 + x_3 &= 0 \\ 0x_1 + 0x_2 + 0x_3 &= 0 \end{aligned}$$

The second equation tells us that $x_3 = 0$. Substituting into the first equation we have,

$$0x_1 + x_2 + 0 = 0$$

This tells us that $x_2 = 0$.

For all 3 equations it doesn't matter what x_1 is since it is always multiplied by 0. That means the solution set is,

$$\left\{ \boldsymbol{x} = \begin{pmatrix} x_1 \\ 0 \\ 0 \end{pmatrix} : x_1 \in \mathbb{R} \right\}$$

We can write this as,

$$x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

All eigenvectors are of this form. So we can choose any value we like as an easy example to get an eigenvector, say, $x_1 = 1$, which means an eigenvector is,

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

But any vector of this form is also an eigenvector. It is equally correct to say that,

$$\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} -7 \\ 0 \\ 0 \end{pmatrix}$$

are eigenvectors.

The eigenspace is the set of all eigenvectors for that particular eigenvalue. That means that,

$$\left\{ \boldsymbol{x} = \begin{pmatrix} x_1 \\ 0 \\ 0 \end{pmatrix} : x_1 \in \mathbb{R} \right\}$$

is the eigenspace for that eigenvalue. We could write is as the span of a basis too:

$$Span\{(1,0,0)\}$$

Both ways of writing imply the same subspace (since we are solving the eigenvalue problem the subspace is known as the eigenspace).