Solutions 1

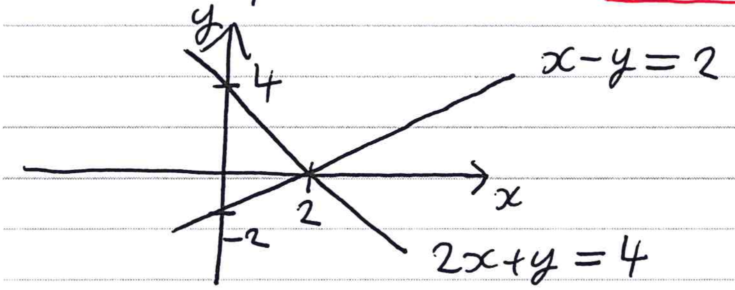
1) $2x+y=4$; $x=0 \Rightarrow y=4$, $y=0 \Rightarrow x=2$

$x-y=2$; $x=0 \Rightarrow y=-2$, $y=0 \Rightarrow x=2$

Straight lines cross at either 1 point, ∞ points, or not at all.

By inspection, the point of intersection is $(2,0)$,

Solution to system

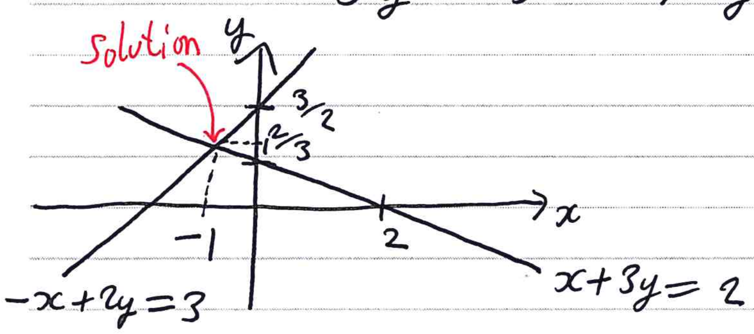


2) $x+3y=2$; Interceptrs are $(0,\frac{2}{3}),(2,0)$

$-x+2y=3$; Intercepts are $(0,\frac{3}{2}),(-3,0)$

Adding the equations gives,

$$5y=5 \Rightarrow y=1 \quad ; \quad x=-1$$



3) $x - y = 1$; Intercepts are $(0,-1)$, $(1,0)$

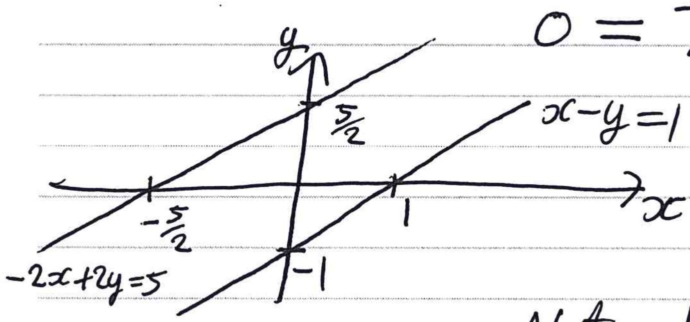
$-2x + 2y = 5$; " $(0, \frac{5}{2})$, $(-\frac{5}{2}, 0)$

$(2 \times$ equation $1)$ + equation 2 gives,

$0 = 7$ $\times$ False statement

$\therefore$ no solution

(no intersection)



Note also that the lines are parallel since $\frac{dy}{dx} = 1$ for both lines,

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4) $\frac{x+3}{4} + \frac{y-1}{3} = 1$; Intercepts are $(0, \frac{7}{4})$, $(\frac{7}{3}, 0)$

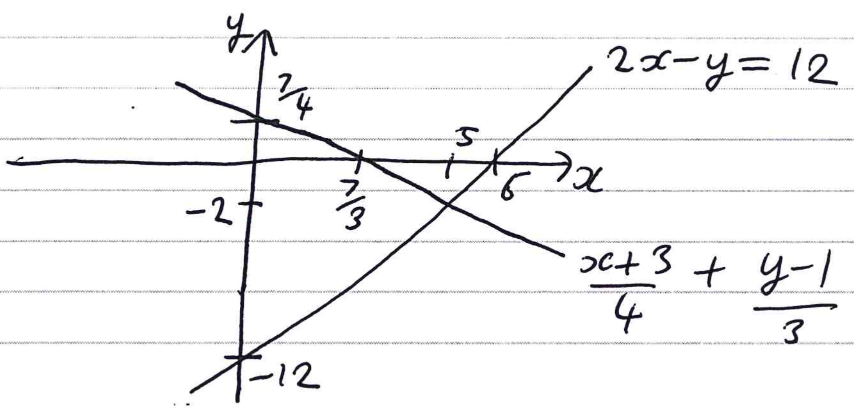
$2x - y = 12$; " $(0,-12)$, $(6,0)$

$12 \times$ equation 1 gives,

$3x + 4y = 7$

Adding this to $4 \times$ equation 2 gives,

$11x = 55 \Rightarrow x = 5 \therefore y = -2$,



5) This matrix is already in echelon form.

With variables x_1, x_2, x_3 (or x, y, z etc.)

the third line gives,

$$x_3 = -1$$

Back substitution:

Row 2: $x_2 - 2x_3 = 1$

$$x_2 - 2(-1) = 1$$

$$x_2 = -1$$

Row 1: $x_1 - x_2 = 3$

$$x_1 - (-1) = 3$$

$$x_1 = 2$$

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6) $$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 3 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & 1 & -1 & 3 \\ 0 & 1 & 2 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -3 & 3 \\ 0 & 1 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 3 & 0 \end{array}\right)$$

Row 3: $x_3 = 0$

Row 2: $x_2 - x_3 = 1$

$$x_2 = 1$$

Row 1: $x_1 - x_2 + x_3 = 0$

$$x_1 = 1$$

7) $\left(\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 1 & -2 & 1 & -2 \\ 1 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 5 & -1 & 4 \\ 0 & 2 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right)$

Row 3: $x_3 = 1$

Row 2: $x_2 = 1$

Row 1: $x_1 - 2x_2 + x_3 = -2$

$x_1 = -1$

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8) $x + 2y = 7$
$2x + y = 8$ $\Longrightarrow \left(\begin{array}{cc|c} 1 & 2 & 7 \\ 2 & 1 & 8 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & -3 & -6 \end{array}\right)$

$\sim \left(\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right)$ $\therefore$ $x = 3$, $y = 2$

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9) $-3x + 5y = -22$
$3x + 4y = 4$
$4x - 8y = 32$ $\Longrightarrow \left(\begin{array}{cc|c} -3 & 5 & -22 \\ 3 & 4 & 4 \\ 4 & -8 & 32 \end{array}\right) \sim \left(\begin{array}{cc|c} 4 & -8 & 32 \\ 3 & 4 & 4 \\ -3 & 5 & -22 \end{array}\right)$

$\sim \left(\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 10 & -20 \\ 0 & -1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right)$

$\therefore$ $x = 4$, $y = -2$

10) $x_1 - 3x_3 = -2$

$3x_1 + x_2 - 2x_3 = 5$ $\Longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{array}\right)$

$2x_1 + 2x_2 + x_3 = 4$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right) \quad \therefore \quad \begin{array}{l} x_1 = 4 \\ x_2 = -3 \\ x_3 = 2 \end{array}$$

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11) $x_1 + x_2 - 5x_3 = 3$

$x_1 - 2x_3 = 1$ $\Longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -5 & 3 \\ 1 & 0 & -2 & 1 \\ 2 & -1 & -1 & 0 \end{array}\right)$

$2x_1 - x_2 - x_3 = 0$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & -1 & 3 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

x_3 is free

Writing $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, the solution is; $\underline{x} = \begin{pmatrix} 1 + 2x_3 \\ 2 + 3x_3 \\ x_3 \end{pmatrix}$

$$= \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

12) $3x+3y+12z=6$

$x+y+4z=2$

$2x+5y+20z=10$

$-x+2y+8z=4$

$$\Longrightarrow \left(\begin{array}{ccc|c} 3 & 3 & 12 & 6 \\ 1 & 1 & 4 & 2 \\ 2 & 5 & 20 & 10 \\ -1 & 2 & 8 & 4 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 12 & 6 \\ 0 & 3 & 12 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 4 & 2 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

z is free

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2-4z \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + z\begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$$

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13) $$\left(\begin{array}{cc|c} 2 & -1 & 3 \\ -4 & 2 & k \\ 4 & -2 & 6 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{3}{2} \\ 4 & -2 & 6 \\ 0 & 0 & k+6 \end{array}\right)$$

Row 3 requires $k=-6$. But check if rows 1 & 2 are consistent!

$$\sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & k+6 \end{array}\right)$$

$\therefore$ consistent for $k=-6$.

14) a) Elementary; $2 \times$ row 2,

b) Elementary; $(2 \times \text{row } 1) + \text{row } 2$,

c) Not elementary; Require 2 row operations,

$\rightarrow 2 \times$ row 1,

$\rightarrow$ Swap rows 2 & 3,

d) Elementary; $-5 \times$ row 2 + row 3,

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15) a) $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \underset{\underline{\underline{E}}_1}{\sim} \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} \underset{\underline{\underline{E}}_2}{\sim} \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} \underset{\underline{\underline{E}}_3}{\sim} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\underline{\underline{E}}_1 = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

$$\underline{\underline{E}}_3 \underline{\underline{E}}_2 \underline{\underline{E}}_1 \underline{\underline{A}} = \underline{\underline{I}} \implies \underline{\underline{A}} = \underline{\underline{E}}_1^{-1} \underline{\underline{E}}_2^{-1} \underline{\underline{E}}_3^{-1}$$

Remembering, $\underline{\underline{A}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \implies \underline{\underline{A}}^{-1} = \dfrac{1}{|\underline{\underline{A}}|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$;

$$\underline{\underline{E}}_1^{-1} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_3^{-1} = -2 \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix},$$

$$\therefore \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$$

b) $\begin{pmatrix} 4 & -1 \\ 3 & -1 \end{pmatrix} \underset{\underline{\underline{E}}_1}{\sim} \begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix} \underset{\underline{\underline{E}}_2}{\sim} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \underset{\underline{\underline{E}}_3}{\sim} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\underline{\underline{E}}_1 = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2 = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\underline{\underline{E}}_1^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2^{-1} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_3^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\therefore \begin{pmatrix} 4 & -1 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

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c) $\begin{pmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \underset{\underline{\underline{E}}_1}{\sim} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \underset{\underline{\underline{E}}_2}{\sim} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\underline{\underline{E}}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2 = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\underline{\underline{E}}_1^{-1}$; $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$

$\underline{\underline{E}}_1^{-1}$

$\underline{\underline{E}}_2^{-1}$;
$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$\underline{\underline{E}}_2^{-1}$

$$\therefore \begin{pmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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16) a)
$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 4 & 0 & 1 & 0 \\ 3 & 6 & 5 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -3 & 1 & 0 \\ 0 & 3 & 2 & -3 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{3}{2} & -\frac{3}{2} & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right)$$

$\therefore$ Inverse is $\begin{pmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{pmatrix}$

b) $\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 7 & -10 & 0 & 1 & 0 \\ 7 & 16 & -21 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & -3 & 1 & 0 \\ 0 & 2 & -14 & -7 & 0 & 1 \end{array}\right)$

$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & -3 & 1 & 0 \\ 0 & 0 & 0 & -1 & -2 & 1 \end{array}\right)$ Can't obtain identity matrix on the left $\therefore$ no inverse.

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17) $\begin{pmatrix} 3 & x \\ -2 & -3 \end{pmatrix}$ is singular if the determinant is 0;

$-9 + 2x = 0 \Rightarrow x = \frac{9}{2}$

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18) a) $\begin{pmatrix} -2 & 1 \\ -6 & 4 \end{pmatrix} \underset{\underline{\underline{E}}}{\sim} \begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix}$

$\underline{\underline{E}} = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}, \quad \underline{\underline{E}}^{-1} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$

$\underline{\underline{A}} = \underline{\underline{L}}\,\underline{\underline{U}}$, but $\underline{\underline{A}} = \underline{\underline{E}}^{-1}\underline{\underline{U}} \therefore \underline{\underline{L}} = \underline{\underline{E}}^{-1}$

$\therefore \underline{\underline{A}} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{pmatrix} \underset{\underline{\underline{E}}_1}{\sim} \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ -3 & 1 & 0 \end{pmatrix} \underset{\underline{\underline{E}}_2}{\sim} \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$

$\underset{\underline{\underline{E}}_3}{\sim} \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix} = \underline{\underline{U}}$

$\underline{\underline{E}}_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

$\underline{\underline{E}}_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

$\underline{\underline{L}} = \underline{\underline{E}}_1^{-1} \underline{\underline{E}}_2^{-1} \underline{\underline{E}}_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$

$$\therefore \begin{pmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

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c) $$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 10 & 12 & 3 \end{pmatrix} \underset{\underline{\underline{E}}_1}{\sim} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 12 & 3 \end{pmatrix} \underset{\underline{\underline{E}}_2}{\sim} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 7 \end{pmatrix} = \underline{\underline{U}}$$

$$\underline{\underline{E}}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

$$\underline{\underline{E}}_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}, \quad \underline{\underline{E}}_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{pmatrix}$$

$$\underline{\underline{L}} = \underline{\underline{E}}_1^{-1}\underline{\underline{E}}_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & -4 & 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 10 & 12 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & -4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 7 \end{pmatrix}$$

19) $$\begin{aligned} 2x + y &= 1 \\ y - z &= 2 \\ -2x + y + z &= -2 \end{aligned} \implies \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ -2 & 1 & 1 \end{pmatrix}$$ Coefficient matrix

$$\underset{E_1}{\sim} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 1 \end{pmatrix} \underset{E_2}{\sim} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix} = U$$

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

$$E_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \quad E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$$

$$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\underline{y} = \underline{\underline{U}}\,\underline{x} \implies \underline{\underline{L}}\,\underline{y} = \underline{\underline{A}}\,\underline{x} \quad (= \underline{b})$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$y_1 = 1$

$y_2 = 2$

$-y_1 + 2y_2 + y_3 = -2 \implies y_3 = -5$

$$\underline{\underline{U}}\,\underline{x} = \underline{y} \implies \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

$x_3 = -\frac{5}{3}$

$x_2 - x_3 = 2 \implies x_2 = \frac{1}{3}$

$2x_1 + x_2 = 1 \implies x_1 = \frac{1}{3}$

$$\therefore \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ -\frac{5}{3} \end{pmatrix}$$